AIAA JOURNAL
Vol. 48, No. 11, November 2010

Technical Notes

Observer for Phased Microphone Array Signal Processing with Nonlinear Output

Xun Huang,* Igor Vinogradov,† Long Bai,‡ and Jianchao Ji§
Peking University, 100871 Beijing, People's Republic of China

DOI: 10.2514/1.J050474

Nomenclature

A	=	dynamics matrix in a time-invariant linear system
A_B	=	acoustic intensities of background noise
A_{BS}	=	acoustic intensities of background noise plus test model sound
A_S	=	acoustic intensities of test model sound
B	=	control input matrices in a time-invariant linear system
c	=	sound speed, m/s
E	=	observer approximation error in frequency domain
e	=	observer approximation error
G	=	output matrices in a time-invariant linear system
j	=	complex part
K	=	observer gain
m	=	the number of microphones in an array
r	=	the distance from microphones to the noise source, m
t	=	time, s
u	=	control input
X, Y	=	the counterparts of x and y in frequency domain
x	=	noise source, $\in R^1$, Pa
y	=	array microphones output, $\in R^m$, Pa
τ	=	time delay, s
ϕ	=	phase shift, rad
ω	=	angular frequency, rad/s
$\langle\,\rangle$	=	arithmetic mean operation

Subscripts

B	=	background noise part
BS	=	background noise and testing model signal
S	=	testing model signal

Superscripts

m	=	mth terms of a Fourier series
T	=	transpose operation
$\dot{}$	=	ordinary differential with respect to time
$\hat{}$	=	observer approximation
$*$	=	complex conjugate
$'$	=	partial differential with respect to time

Received 5 February 2010; revision received 17 April 2010; accepted for publication 10 August 2010.

*Associate Professor, State Key Laboratory of Turbulence and Complex Systems, Department of Mechanics and Aerospace Engineering, College of Engineering; Visiting Researcher, University of Southampton, Aeronautics and Astronautics, Southampton, England S0171BJ, United Kingdom; huangxun@pku.edu.cn. Senior Member AIAA.

†Research Fellow, Department of Mechanics and Aerospace Engineering, College of Engineering; ivinogra@pku.edu.cn.

‡Research Student, Department of Mechanics and Aerospace Engineering, College of Engineering; bailong211@pku.edu.cn.

§Research Student, Department of Mechanics and Aerospace Engineering, College of Engineering.

I. Introduction

THE nuisance noise generated from aircraft has significant environmental impacts on communities near an airport. The noise is produced by the complex interaction of aerodynamic surfaces and the surrounding turbulence flow [1]. The distribution of noise sources on the body has to be correctly identified before any development of an efficient noise control strategy can take place. Microphone arrays [2] are being increasingly used in the aerospace industry [3] to determine the noise source distributions. Various array processing algorithms [4], such as conventional beamforming [4] and robust adaptive beamforming [5], have been proposed. The former is still more prevalent for its robust performance in a noisy aeroacoustic testing environment.

The number of array microphones continues to increase to improve the resolution and to avoid spatial aliasing [6]. However, for a conventional beamforming algorithm, it is extremely difficult to process the huge amount of data from a large capacity array in real time. Recently, a new approach with real-time computational capability has been proposed [7]. This is the so-called observer-based algorithm capable of generating acoustic images recursively over sampling data. This real-time algorithm is quite helpful for practical applications, especially for field tests, since any defects in the onsite experiments can be discovered instantaneously.

The observer-based algorithm is further extended in this work, particularly for a case of coherent noise sources. The objective of this note is to present the theoretical background and summarize the related mathematical equations. The Note is organized as follows. The typical beamforming algorithm and its limitations are described in Sec.II. Section III summarizes the observer theory in classical control. An improved observer-based algorithm is developed in Sec. IV. A summary of the work is presented in Sec. V.

II. Typical Beamforming Formulations

For a microphone array with m microphones, the output y denotes all microphones measurements, $y \in R^{m\times 1}$. With the assumption of a single noise source $x \in R^1$ in a free sound propagation space, using a solution of the Green's function for the wave equation,

$$y(t) = \frac{1}{4\pi r} x(t-\tau), \qquad \tau = \frac{r}{c} \tag{1}$$

where c is the speed of sound, and $r \in R^{m\times 1}$ is the distance between the noise source x and each microphone. Generally, for most aeroacoustic applications, beamforming is conducted in the frequency domain [8]. The frequency domain version of Eq. (1) is

$$Y(\omega) = \frac{1}{4\pi r} X(\omega) e^{-j\omega\tau} \tag{2}$$

One detrimental factor in aeroacoustic measurements comes from background noise, particularly for a closed-section wind tunnel. To take into account the effect of the background noise, which can be generated by a tunnel drive or from a coherent noise nearby, Eq. (2) is modified to

$$Y_B = \frac{1}{4\pi r} X_B e^{-j\omega\tau} \tag{3}$$

$$Y_{BS} = \frac{1}{4\pi r} X_{BS} e^{-j\omega\tau} \tag{4}$$

where the subscript of B denotes the experiments without the installment of any model. From here, the symbol ω is omitted from the rest of the text. The measurement Y_B comes solely from background noise. The subscript of BS denotes the experiments with the presence of a model, and Y_{BS} records the combined effect of the sound from the test model and noise from the testing facility.

The following operations can be conducted to approximate the testing model sound X:

$$A_B = Y_B Y_B^*, \qquad A_{BS} = Y_{BS} Y_{BS}^* \tag{5}$$

$$\langle A_S \rangle = \left(\frac{1}{4\pi r}\right)^2 \langle X_S X_S^* \rangle \approx \langle A_{BS} \rangle - \langle A_B \rangle \tag{6}$$

where the superscript of $*$ denotes the complex conjugate, and $\langle\,\rangle$ denotes the arithmetic mean. The averaged sound source $\langle X_S X_S^* \rangle$ can be straightforwardly obtained from Eq. (6). The operations in Eqs. (5) and (6) can be conducted repeatedly over every scanned point to generate a sound pressure image, which can help to identify the position of the dominant noise source. More details of the typical beamforming algorithm and its implementations can be found in [3,6].

A couple of assumptions are implicitly made in Eq. (6). First, the signal of interest is assumed almost stationary and, therefore, can be approximated by averaging over blocks ($\langle\,\rangle$). Second, there is little coherence between X_B and X_S. Therefore, $\langle X_B X_S^* \rangle = 0$, $\langle X_B^* X_S \rangle = 0$, and the approximation in Eq. (6) is valid. However, the time-consuming operation of ($\langle\,\rangle$) prevents identifying the noise source in real time, while the assumption of little coherence could be invalid in some cases of coherent or extended sources.

III. Classical Observer Formulations

Recently, a new algorithm has been proposed by our group. The idea is derived from observer theory in classical control. This section provides a brief introduction into the concept, while the details can be found in any linear control textbook [9]. First, a time-invariant linear system can be represented in continuous time in the following form:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{7}$$

$$y(t) = Gx(t) \tag{8}$$

where x is the state vector, the symbol of $\dot{}$ denotes $\mathrm{d}/\mathrm{d}t$, y is the output, u the control input, A is the state matrix, B is the input matrix, and G is the output matrix. Equation (7) describes the dynamics of x. Equation (8) is the measurement equation, which has a form similar to Eq. (2) for $G = 1/(4\pi r)e^{-j\omega\tau}$. However, it is worthwhile tp emphasize that Eqs. (7) and (8) are in the time domain.

A classical state observer (so-called Luenberger observer) to approximate x from y is

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y - \hat{y}) \tag{9}$$

$$\hat{y}(t) = G\hat{x}(t) \tag{10}$$

where $\hat{x}$ is the approximation of x, and K is the observer gain. The estimation error is $e \triangleq x - \hat{x}$, and its dynamics are

$$\dot{e} = (A - KG)e \tag{11}$$

which is obtained by calculating the difference between Eqs. (7) and (9). It can be shown that e converges to zero when $t \to \infty$, as long as the all the eigenvalues of the matrix $(A - KG)$ have negative real parts.

IV. New Observer-Based Algorithm

The goal of this work is to develop a new observer-based algorithm, which has real-time capability, even for correlated noise distributions. Although the aforementioned observer is formulated in the time domain, the classical beamforming is generally conducted in the frequency domain. To reach a consistent form, the array measurements and the sound source are represented by the Fourier series. All variables in the time domain can be expressed by

$$y(t) = \sum_{m=-\infty}^{\infty} Y_m e^{jmt}, \qquad x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jmt} \tag{12}$$

$$Y_m = \frac{1}{4\pi r} X_m e^{-jm\tau} \tag{13}$$

X_m and Y_m consists of a time-invariant linear system, which can be expressed in the following form:

$$\dot{X}_m = AX_m, \qquad Y_m = GX_m \tag{14}$$

where $G = e^{-jm\tau}/(4\pi r)$ is the sound propagation vector in a free space, and the subscript of m can be omitted without loss of generality. It is easy to see that the corresponding observer has the form of

$$\dot{\hat{X}} = A\hat{X} + K(Y - \hat{Y}), \qquad \hat{Y} = G\hat{X} \tag{15}$$

For practical aeroacoustic measurements conducted in a closed-section wind tunnel, $X = [X_B, X_S, \phi]$ and, as explained previously, $Y = [Y_B, Y_{BS}]$. The two variables of X and Y satisfy

$$\begin{bmatrix} Y_B \\ Y_{BS} \end{bmatrix} = \begin{bmatrix} G_B & \\ G_B e^{j\phi} & G_S \end{bmatrix} \begin{bmatrix} X_B \\ X_S \end{bmatrix} \tag{16}$$

where G_B and G_S have a form similar to G, and ϕ is the phase difference between the measurements of Y_B and Y_{BS}, which were assumed already known in previous work by Huang [7]. A Luenberger observer [Eq. (9)] can be designed accordingly to approximate X_B and X_S recursively. More details and discussions of real-time performance can be found in [7].

The ϕ can be approximated in practical applications by checking the correlation between Y_B and Y_{BS}. However, it is sometimes impossible to achieve the desired accuracy. A more generalized algorithm is proposed in this work to address this issue. As a result of unknown ϕ, the relationship between Y and X is not linear anymore. It can be written by

$$\begin{bmatrix} \dot{X}_B \\ \dot{X}_S \\ \dot{\phi} \end{bmatrix} = A \begin{bmatrix} X_B \\ X_S \\ \phi \end{bmatrix}, \qquad \begin{bmatrix} Y_B \\ Y_{BS} \end{bmatrix} = G(X_B, X_S, \phi) \tag{17}$$

For problems with nonlinear output of $G(X_B, X_S, \phi)$, a new observer is proposed here as

$$\begin{aligned} \dot{\hat{X}} &= A\hat{X} + KG'^*(\hat{X}_B, \hat{X}_S, \hat{\phi})(Y - \hat{Y}) \\ &= A\hat{X} + KG'^*(\hat{X}_B, \hat{X}_S, \hat{\phi})[G(X_B, X_S, \phi) - G(\hat{X}_B, \hat{X}_S, \hat{\phi})] \end{aligned} \tag{18}$$

where the symbol of $*$ is the complex conjugate, and $G' \triangleq \partial G/\partial X$, which is

$$\begin{bmatrix} \frac{\partial Y_B}{\partial X_B} & \frac{\partial Y_B}{\partial X_S} & \frac{\partial Y_B}{\partial \phi} \\ \frac{\partial Y_{BS}}{\partial X_B} & \frac{\partial Y_{BS}}{\partial X_S} & \frac{\partial Y_{BS}}{\partial \phi} \end{bmatrix} = \begin{bmatrix} G_B & 0 & 0 \\ G_B e^{j\phi} & G_S & jG_B e^{j\phi} X_B \end{bmatrix} \tag{19}$$

The estimation error is $E \triangleq (X_B, X_S, \phi)^T - (\hat{X}_B, \hat{X}_S, \hat{\phi})^T$, where the subscript T is the transpose of a vector. The estimation error dynamics (with respect to the sampling data block number) are nonlinear but can be linearized around zero by a Taylor series expansion:

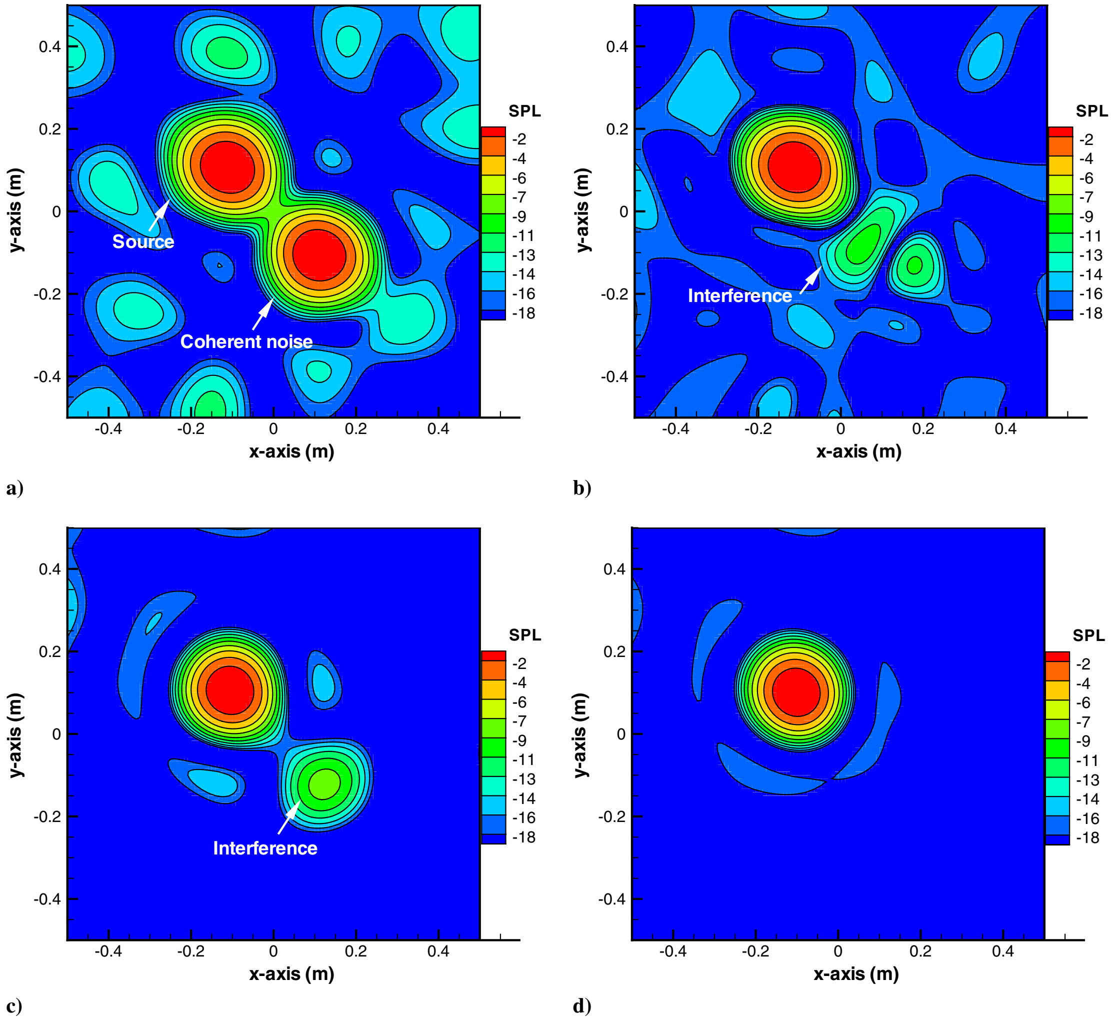

Fig. 1 Acoustic images of sound pressure level contours in decibels: a) two coherent sources, b) background noise removed by conventional beamforming, and c) and observer results at the 100th block and d) the 2000th block (SPL denotes sound pressure level).

$$\dot{E} = [A - KG'^{*}(\hat{X}_B, \hat{X}_S, \hat{\phi})G'(\hat{X}_B, \hat{X}_S, \hat{\phi})]E \tag{20}$$

As $G'^{*}(\hat{X}_B, \hat{X}_S, \hat{\phi})G'(\hat{X}_B, \hat{X}_S, \hat{\phi})$ is a positive definite Hermitian matrix, the error dynamics are locally asymptotically stable, which can be proved using the Lyapunov stability theory. In other words, the error E approaches zero while Eqs. (18) and (19) are recursively computed over sampling data blocks of Y_B and Y_{BS}. A detailed derivation and the guidance of the selection of K can be found in [10].

In summary, the observer-based approach can be conducted in the following steps: 1) prepare the invariant matrix A and matrix G; 2) make an initial guess of $\hat{X} = (\hat{X}_B, \hat{X}_S, \hat{\phi})$, where a small difference between ϕ and $\hat{\phi}$ is preferred for asymptotic stability; 3) compute $Y = (Y_B, Y_{BS})$ iteratively for each sampling data block; and 4) conduct Eqs. (18) and (19) recursively over sampling data blocks to update the approximation $\hat{X}$. The working and benefits of the algorithm are demonstrated in Fig. 1. Two coherent monopole sources of the identical strength are shown in Fig. 1a, which collectively produce the Y_{BS} at a microphone array of 56 sensors. In addition, it is assumed that Y_B of the coherence noise can be measured separately. The source of the interest X_S [the top left graph in Fig. 1a] is approximated by conventional beamforming and the observer-based algorithm. Figure 1b shows the estimation of X_S by conventional beamforming that is averaged over 100 blocks. It can be seen that the beamforming outcome is detrimentally affected by the coherent noise. Figure 1c shows the recursive result of the observer-based algorithm at the 100th block. The interference from the coherent background noise is still visible, but the side effect, particularly on sidelobes, is reduced. Figure 1d shows the recursive result of the observer-based algorithm at the 2000th block. It can be seen that the interference from the coherent noise is almost completely suppressed, and the sound of interest is satisfactorily restored. In this case, $\phi = 1.0134$ rad, while its initial guess $\hat{\phi} = 1.5$. Figure 2 shows that the approximation error of ϕ quickly converges to zero in the iterations. Finally, it is worthwhile to mention that aeroacoustic images are normally generated separately at various single frequencies. The results presented in Figs. 1 and 2 are at

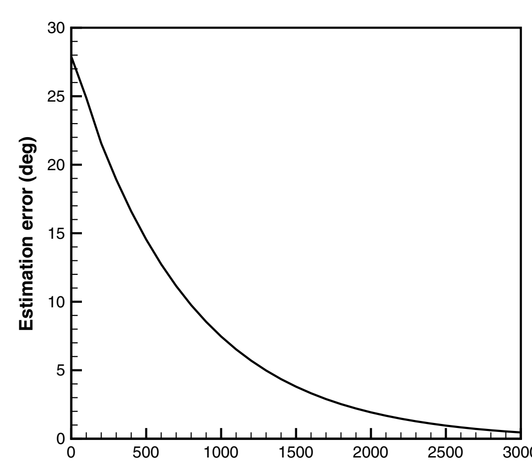

Fig. 2 The convergence of the estimation error $(\phi - \hat{\phi})$ of each sampling data block.

$f = 3$ kHz. The same working procedure of the aforementioned observer-based algorithm can be followed straightforwardly to achieve results at other frequencies.

V. Conclusions

A new approach has been proposed as an alternative of classical beamforming. The corresponding signal processing can be conducted in real time because the observer-based algorithm is recursive, and the defects in measurements can be detected instantaneously and rectified accordingly. In addition, and maybe more important, the algorithm can identify coherent noise sources. Compared with classical beamforming, the incoherence assumption is not present in the new approach, which is especially valuable and interesting for array signal processing. The other assumptions generally adopted in the beamforming methods, such as a free space of sound propagation, are still accepted in this observer-based method. It is important to note that the beam patterns of both classical beamforming and the observer-based algorithm are comparable. Related discussion, along with the investigation of convergence error and speed, can be found in [7]. Advanced signal-processing techniques can be employed to further improve the resolution and accuracy of classical beamforming, as well as the observer-based algorithm. Those algorithms have expensive computational costs and can only be conducted offline. Interested readers can refer to the literature [8,11,12].

Acknowledgment

This work was supported by the National Science Foundation grant of China (90916003 and 11050110109).

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N. Chokani
Associate Editor